

Example 1

A class of 28 people collected the following data, which represent their heights x and arm spans y (rounded to the nearest inch).

(60, 61), (65, 65), (68, 67), (72, 73), (61, 62), (63, 63), (70, 71),
(75, 74), (71, 72), (62, 60), (65, 65), (66, 68), (62, 62), (72, 73),
(70, 70), (69, 68), (69, 70), (60, 61), (63, 63), (64, 64), (71, 71),
(68, 67), (69, 70), (70, 72), (65, 65), (64, 63), (71, 70), (67, 67),

Find a linear model to represent these data.

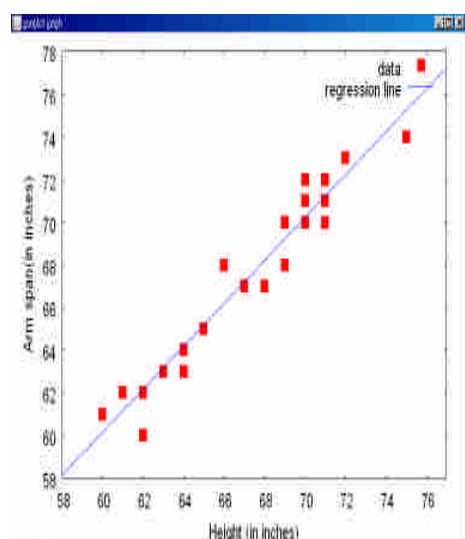
Sol:

There are different ways to model these data with an equation. The simplest would be to observe that x and y are about the same and list the model as simply $y=x$. A more careful analysis would be to use a procedure from statistics called linear regression. (You will study this procedure in Section 13.9.) The least squares regression line for these data is

$$y = 1.006x - 0.23$$

Least squares regression line

The graph of model and the data are shown in Figure P.32. From this model, you can see that a person's arm span tends to be about the same as his or her height.



```
(%i32) xy:[[60, 61], [65, 65], [68, 67], [72, 73], [61, 62], [63, 63], [70, 71],  
[75, 74], [71, 72], [62, 60], [65, 65], [66, 68], [62, 62], [72, 73],  
[70, 70], [69, 68], [69, 70], [60, 61], [63, 63], [64, 64], [71, 71],  
[68, 67], [69, 70], [70, 72], [65, 65], [64, 63], [71, 70], [67, 67]]$
```

```
(%i45) plot2d([discrete, xy], [style,points])$
```



```
(%i7) plot2d([1.006*x-0.23],[x,50,80]);
```

```
(%o7)
```

```
(%i50) plot2d([[discrete,xy], 1.006*x-0.23], [x,60,77],  
[style, [points,5,2,6], [lines,1,1]],  
[legend,"data","regression line"],  
[xlabel,"Height (in inches)", [ylabel,"Arm span(in inches)"]])$
```

Example 2

A basketball is dropped from a height of about $5\frac{1}{4}$ feet. The height of the basketball is recorded 23 times at intervals of about 0.02 second. The results are shown in the table.

Time	0.0	0.02	0.04	0.06	0.08	0.099996
Height	5.23594	5.20353	5.16031	5.0991	5.02707	4.95146

Time	0.119996	0.139992	0.159988	0.179988	0.199984	0.219984
Height	4.85062	4.74979	4.63096	4.50132	4.35728	4.09523

Time	0.23998	0.25993	0.27998	0.299976	0.319972	0.339961
Height	4.02958	3.84593	3.65507	3.44981	3.23375	3.01048

Time	0.359961	0.379951	0.399941	0.419941	0.439941
Height	2.76921	2.52074	2.25786	1.98058	1.63488

Find a model to fit these data. Then use the model to predict the time when the basketball will hit the ground.

Sol:

Begin by drawing a scatter plot of the data, as shown in Figure P.33. From the scatter plot, you can see that the data do not appear to be linear. It does appear, however, that they might be quadratic. To check this, enter the data into a calculator or computer that has a quadratic regression program. You should obtain the model

$$s = -15.45t^2 - 1.30t + 5.234 \quad \text{Least squares regression quadratic}$$

Using this model, you can predict the time when the basketball hits the ground by substituting 0 for s and solving the resulting equation for t.



$$0 = -15.45t^2 - 1.30t + 5.234$$

Let $s=0$

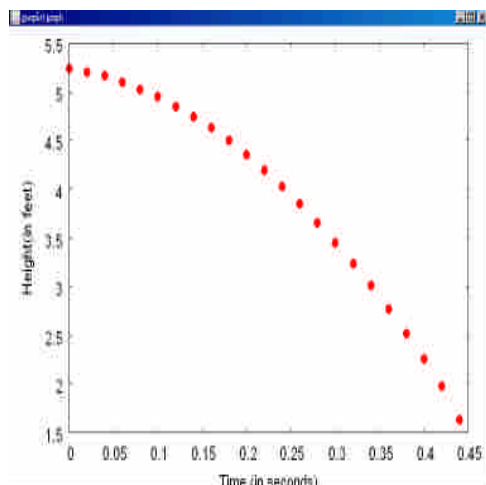
$$t = \frac{1.30 \pm \sqrt{(-1.30)^2 - 4(-15.45)(5.234)}}{2(-15.45)}$$

Quadratic Formula

$$t \approx 0.54$$

Choose positive solution.

The solution is about 0.54 second. In other words, the basketball will continue to fall for about 0.1 second more before hitting the ground.



```
(%i1) xy:[[0, 5.23594], [0.02, 5.20353], [0.04, 5.16031], [0.06, 5.0991], [0.08, 5.02707],  
[0.099996, 4.95146], [0.119996, 4.85062],  
[0.139992, 4.74979], [0.159988, 4.63096], [0.179988, 4.50132], [0.199984, 4.35728],  
[0.219984, 4.19523], [0.23998, 4.02958], [0.25993, 3.84593],  
[0.27998, 3.65507], [0.299976, 3.44981], [0.319972, 3.23375], [0.339961, 3.01048],  
[0.359961, 2.76921], [0.379951, 2.52074], [0.399941, 2.25786],  
[0.419941, 1.98058], [0.439941, 1.63488]]$
```

```
(%i7) plot2d([discrete,xy], [style, [points,4,2,1]],  
[xlabel,"Time (in seconds)"], [ylabel,"Height(in feet)"])$
```

```
(%i1) xy:[[0, 5.23594], [0.02, 5.20353], [0.04, 5.16031], [0.06, 5.0991], [0.08, 5.02707],  
[0.099996, 4.95146], [0.119996, 4.85062],  
[0.139992, 4.74979], [0.159988, 4.63096], [0.179988, 4.50132], [0.199984, 4.35728],  
[0.219984, 4.19523], [0.23998, 4.02958], [0.25993, 3.84593],  
[0.27998, 3.65507], [0.299976, 3.44981], [0.319972, 3.23375], [0.339961, 3.01048],
```



<http://www.npue.edu.tw/academic/math/index.htm>

```
[0.359961, 2.76921], [0.379951, 2.52074], [0.399941, 2.25786],  
[0.419941, 1.98058], [0.439941, 1.63488]]$
```

```
(%i7) plot2d([discrete,xy], [style, [points,4,2,1]],  
[xlabel,"Time (in seconds)", ylabel,"Height(in feet)"])$
```

```
(%i8) solve([-15.45*t^2-1.30*t+5.234=0],[t]);
```

```
rat: replaced 5.234 by 2617/500 = 5.234  
rat: replaced -1.3 by -13/10 = -1.3  
rat: replaced -15.45 by -309/20 = -15.45
```

```
(%o8) [ t =  $-\frac{11\sqrt{6718} + 65}{1545}$ , t =  $\frac{11\sqrt{6718} - 65}{1545}$  ]
```

```
(%i9) float(%);
```

```
(%o9) [ t = -0.62562943117076, t = 0.54148703634876 ]
```

Example 3

The number of hours of daylight on Earth depends on the latitude and the time of year. Here are the numbers of minutes of daylight at a location of 20° N latitude on the longest and shortest days of the year: June 21, 801 minutes; December 22, 655 minutes. Use these data to write a model for the amount of daylight d (in minutes) on each day of the year at a location of 20° N latitude. How could you check the accuracy of your model?

Sol:

Here is one way to create a model. You can hypothesize that the model is a sine function whose period is 365 days. Using the given data, you can conclude that the amplitude of the graph is $(801-655)/2$, or 73. So, one possible model is

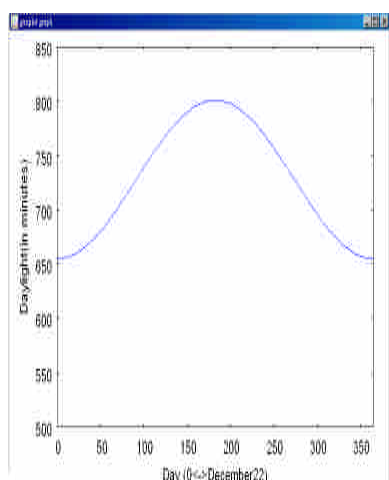
$$d = 728 - 73 \sin\left(\frac{2\pi t}{365} + \frac{\pi}{2}\right)$$

In this model, t represents the number of each day of the year, with December 22 represented by $t=0$. A graph of this model is shown in Figure P.34. To check the accuracy of this model, we used a weather almanac to find the numbers of minutes of daylight on different days of the year at the location of 20° N latitude.



Date	Value of t	Actual Daylight	Daylight Given by Model
Dec 22	0	655 min	655 min
Jan 1	10	657 min	656 min
Feb 1	41	676 min	672 min
Mar 1	69	705 min	701 min
Apr 1	100	740 min	739 min
May 1	130	772 min	773 min
Jun 1	161	796 min	796 min
Jun 21	181	801 min	801 min
Jul 21	191	799 min	800 min
Aug 1	222	782 min	785 min
Sep 1	253	752 min	754 min
Oct 1	283	718 min	716 min
Nov 1	314	685 min	681 min
Dec 1	344	661 min	660 min

You can see that the model is fairly accurate.



```
(%i13) f(t):=728-73*sin(2*%pi*t/365+%pi/2);
```

```
plot2d(f,[t,0,365],[y,500,850],
```

```
[xlabel,"Day (0<->December22)"], [ylabel,"Daylight(in minutes)"])$
```

```
(%o13) f(t):=728-73 sin\left(\frac{2 \pi t}{365} + \frac{\pi}{2}\right)
```