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Example 1

For the function f defined by $f(x)=x^2+7$, evaluate each expression.

a. $f(3a)$ b. $f(b-1)$ c. $\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$, $\Delta x \neq 0$

Sol:

a. $f(3a)=(3a)^2+7$ Substitute 3a for x.

$=9a^2+7$ Simplify.

b. $f(b-1)=(b-1)^2+7$ Substitute b-1 for x.

$=b^2-2b+1+7$ Expand binomial.

$=b^2-2b+8$ Simplify.

c. $\dfrac{f(x+\Delta x)f(x)}{\Delta x}=\dfrac{[(x+\Delta x)^2+7]-(x^2+7)}{\Delta x}$

$$=\frac{x^2+2x\Delta x+(\Delta x)^2+7-x^2-7}{\Delta x}$$

$$=\frac{2x\Delta x+(\Delta x)^2}{\Delta x}$$

$$=\frac{\Delta x(2x+\Delta x)}{\Delta x}$$

$$=2x+\Delta x, \qquad \Delta x \neq 0$$

```
(%i9) f(x):=x^2+7;
(%o9) f(x):=x^2+7

(%i10) f(3*a);
(%o10) 9 a^2+7

(%i11) f(b-1);
(%o11) (b-1)^2+7

(%i15) expand((b-1)^2+7);
(%o15) b^2-2 b+8
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(%i13) [f(x+delta(x))-f(x)]/delta(x);

(%o13) $\left[\frac{(\delta(x)+x)^2-x^2}{\delta(x)}\right]$

(%i16) expand([((delta(x)+x)^2-x^2)/delta(x)]);

(%o16) $[\delta(x)+2x]$

Example 2

a. The domain of the function

$$f(x)=\sqrt{x-1}$$

is the set of all x-values for which $x-1\geq 0$, which is the interval $[1,\infty)$. To find the range observe that $f(x)=\sqrt{x-1}$ is never negative. So, the range is the interval $[0,\infty)$, as indicated in Figure P.23(a).

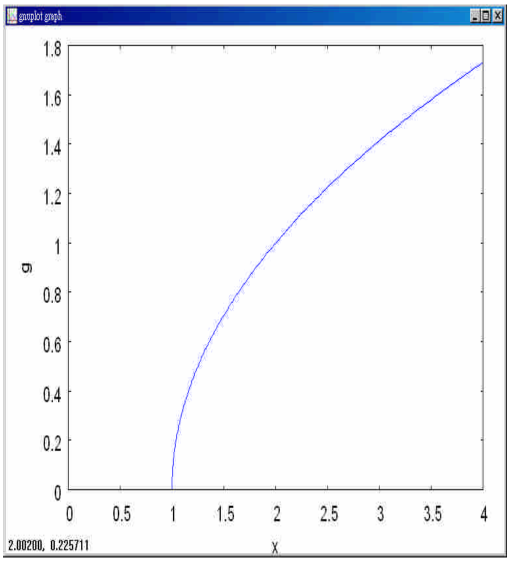
b. The domain of the tangent function, as shown in Figure P.23(b),

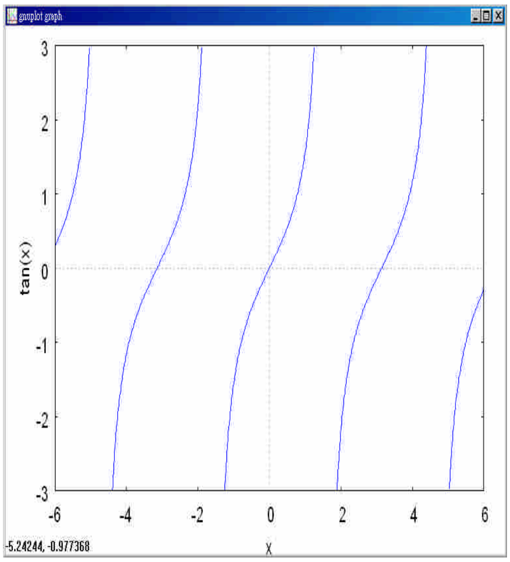
$$f(x)=\tan x$$

is the set of all x-values such that

$x\neq\frac{\pi}{2}+n\pi$, n is an integer. Domain of tangent function

The range of this function is the set of all real numbers. For a review of the characteristics of this and other trigonometric functions, see Appendix D.





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(%i32) g(x):=sqrt(x-1);
plot2d(g,[x,0,4]);

(%o32) $g(x):=\sqrt{x-1}$

(%i31) plot2d (tan(x), [x, -6, 6],[y,-3,3]);

plot2d: some values were clipped.

(%o31)

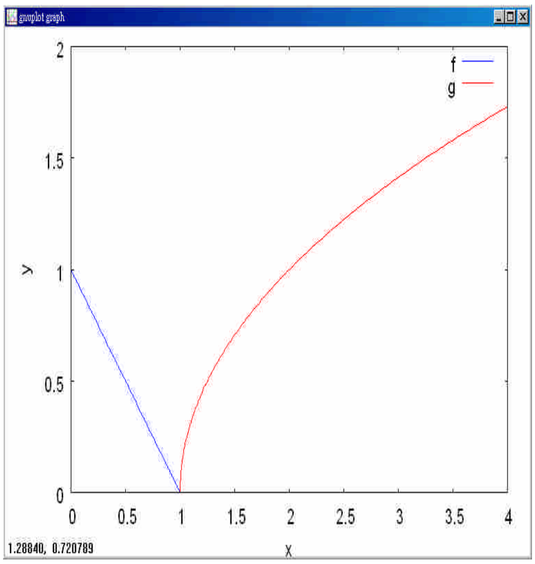
Example 3

Determine the domain and range of the function.

$$f(x)=\begin{cases}1-x, & if\ \ x<1\\ \sqrt{x-1}, & if\ \ x\geq 1\end{cases}$$

Sol:

Because f is defined for $x<1$ and $x\geq 1$, the domain is the entire set of real numbers. On the portion of the domain for which $x\geq 1$, the function behaves as in Example 2(a). For $x<1$, the values of $1-x$ are positive. So, the range of the function is the interval $[0,\infty)$. (See Figure P.24.)



(%i26) f(x):=1-x;
g(x):=sqrt(x-1);

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plot2d([f,g],[x,0,4],[y,0,2]);

(%o26) f(x):=1−x

(%o27) g(x):=$\sqrt{x-1}$

```
plot2d: some values were clipped.
plot2d: expression evaluates to non-numeric value somewhere in pl
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(%o28)

Example 4

Given $f(x)=2x-3$ and $g(x)=\cos x$, find each composite function.

a. $f\circ g$ b. $g\circ f$

Sol:

a. $(f\circ g)(x)=f(g(x))$	Definition of $f\circ g$
$=f(\cos x)$	Substitute $\cos x$ for $g(x)$
$=2(\cos x)-3$	Definition of $f(x)$
$=2\cos x-3$	Simplify.
b. $(g\circ f)(x)=g(f(x))$	Definition of $g\circ f$
$=g(2x-3)$	Substitute $2x-3$ for $f(x)$
$=\cos(2x-3)$	Definition of $g(x)$

Note that $(f\circ g)(x)\neq(g\circ f)(x)$.

(%i64) f(x):=2*x-3;
g(x):=cos(x);

(%o64) f(x):=2 x−3

(%o65) g(x):=cos(x)

(%i67) f(g(x));

(%o67) 2 cos(x)−3

(%i66) g(f(x));

(%o66) cos(2 x−3)

Example 5

Determine whether each function is even, odd, or neither. Then find the zeros of the function.

a. $f(x) = x^3 - x$ b. $g(x) = 1 + \cos x$

Sol:

a. This function is odd because

$f(-x) = (-x)^3 - (-x) = -x^3 + x = -(x^3 - x) = -f(x)$.

The zero of f are found as shown.

$x^3 - x = 0$	Let $f(x) = 0$
$x(x^2 - 1) = x(x-1)(x+1) = 0$	Factor.
$x = 0, 1, -1$	Zero of f

See Figure P.31(a).

b. This function is even because

$g(-x) = 1 + \cos(-x) = 1 + \cos x = g(x)$. $\cos(-x) = \cos(x)$

The zeros of g are found as shown.

$1 + \cos x = 0$	Let $g(x) = 0$
$\cos x = -1$	Subtract 1 from each side.
$x = (2n+1)\pi$, n is an integer.	Zeros of g

See Figure P.31(b)

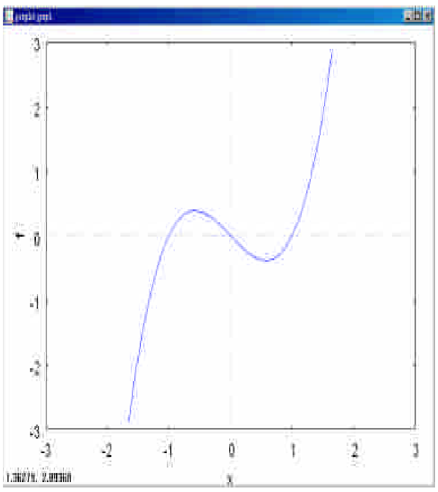

Figure P.31(a)

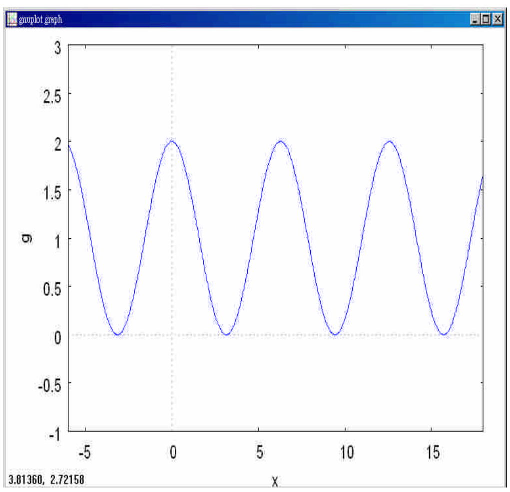

Figure P.31(b)

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(%i47) f(x):=x^3-x;

(%o47) $f(x):=x^3-x$

(%i49) plot2d(f,[x,-3,3],[y,-3,3]);

plot2d: some values were clipped.

(%o49)

(%i60) g(x):=1+cos(x);

(%o60) $g(x):=1+\cos(x)$

(%i62) plot2d(g,[x,-6,18],[y,-1,3]);

(%o62)