Example 1

Find an equation of the line that has a slope of 3 and passes through the point (1, -2).

Sol:

$y - y_1 = m(x - x_1)$ Point slope form

$y - (-2) = 3(x - 1)$ Substitute -2 for y_1, 1 for x_1, and 3 for m.

$y + 2 = 3x - 3$ Simplify.

$y = 3x - 5$ Solve for y.

(%i9) f(m):=m*(x-x1)+y1;

(%o9) f(m):=m(x-x1)+y1

(%i10) f(3);

(%o10) y1+3(x-x1)

(%i11) f(x1,y1):=y1+3*(x-x1);

(%o11) f(x1,y1):=y1+3(x-x1)

(%i12) f(1,-2);

(%o12) 3(x-1)-2

Example 2

a. The population of Kentucky was 3687000 in 1990 and 4042000 in 2000. Over this 10-year period, the average rate of change of the population was

$$Rate \quad of \quad change = \frac{change \ in \ population}{change \ in \ years}$$

$$= \frac{4042000 - 3687000}{2000 - 1990}$$

$$= 35500 \ people \ per \ year.$$

If Kentucky's population continues to increase at this same rate for the next 10 years, it will have a 2010 population of 4937000(see Figure P.16).

b. In tournament water-ski jumping, the ramp rises to a height of 6 feet on a raft that is 21 feet long, as shown in Figure P.17. The slope of the ski ramp is the ratio of its height (the rise) to the length of its base (the run).

$$Slope\ of\ \ ramp = \frac{rise}{run}$$ Rise is vertical change, run is horizontal change.

$$= \frac{6\,feet}{21\,feet}$$

$$= \frac{2}{7}$$

In this case, note that the slope is a ratio and has units.

Example 3

Sketch the graph of each equation.

a. $y = 2x + 1$ b. $y = 2$ c. $3y + x - 6 = 0$

Sol:

a. Because b=1, the y-intercept is (0, 1). Because the slope is m=2, you know that the line rises two units for each unit it moves to the right, as shown in Figure P.18(a).

b. Because b=2, the y-intercept is (0, 2). Because the slope is m=0, you know that the line is horizontal, as shown in Figure P.18(b).

c. Begin by writing the equation in slope-intercept form.

$$3y + x - 6 = 0$$ Write original equation.

$$3y = -x + 6$$ Isolate y-term on the left.

$$y = -\frac{1}{3}x + 2$$ Slope-intercept form

In this form, you can see that the y-intercept is (0, 2) and the slope is $m = -\frac{1}{3}$. This means that the line falls one unit for every three units it moves to the right, as shown in Figure P.18(c).

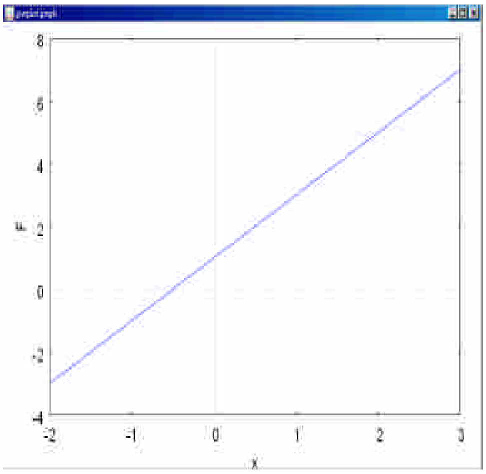


Figure P.18(a)

(%i17) F(x) := 2*x+1;
plot2d(F,[x,-2,3]);

(%o17) F(x):=2 x+1

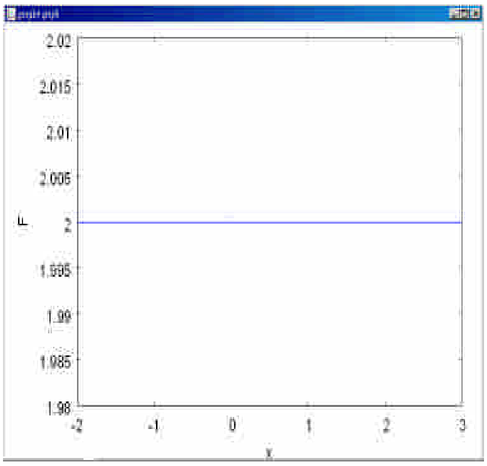


Figure P.18(b)

(%i48) F(x) := 2;
plot2d(F,[x,-1,3]);

(%o48) F(x):=2
(%o49)

http://www.npue.edu.tw/academic/math/index.htm

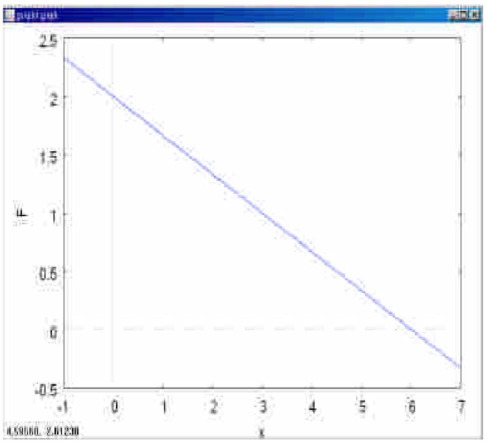


Figure P.18(c)

(%i50) F(x):=-1/3*x+2;
plot2d(F,[x,-1,7]);

(%o50) $F(x):=\frac{-1}{3}x+2$

(%o51)

Example 4

Find the general forms of the equations of the lines that pass through the point (2, -1) and are

a. parallel to the line 2x-3y=5   b. perpendicular to the line 2x-3y=5.

(See Figure P.20.)

Sol:

By writing the linear equation 2x-3y=5 in slope-intercept form, $y=\frac{2}{3}x-\frac{5}{3}$,you can see that the given line has a slope of $m=\frac{2}{3}$.

a. The line through (2, -1) that is parallel to the given line also has a slope of $\frac{2}{3}$.

$y-y_1=m(x-x_1)$ Point-slope form

$y-(-1)=-\frac{3}{2}(x-2)$ Substitute.

$2(y+1)=-3(x-2)$ Simplify.

$3x+2y-4=0$ General form

(%i52) f(m):=m*(x-x1)+y1;

(%o52) $f(m):=m(x-x1)+y1$

(%i53) f(2/3);

(%o53) $y1+\frac{2(x-x1)}{3}$

(%i54) f(x1,y1):=y1+(2/3)*(x-x1);

(%o54) $f(x1,y1):=y1+\frac{2}{3}(x-x1)$

(%i55) f(2,-1);

(%o55) $\frac{2(x-2)}{3}-1$

b. Using the negative reciprocal of the slope of the given line, you can determine that the slope of a line perpendicular to the given line is $-\frac{3}{2}$. So, the line through the point (2, -1) that is perpendicular to the given line has the following equation.

$y-y_1=m(x-x_1)$	Point-slope form
$y-(-1)=-\frac{3}{2}(x-2)$	Substitute.
$2(y+1)=-3(x-2)$	Simplify.
$3x+2y-1=0$	General form

(%i56) f(m):=m*(x-x1)+y1;

(%o56) $f(m):=m(x-x1)+y1$

http://www.npue.edu.tw/academic/math/index.htm

(%i57) f(-3/2);

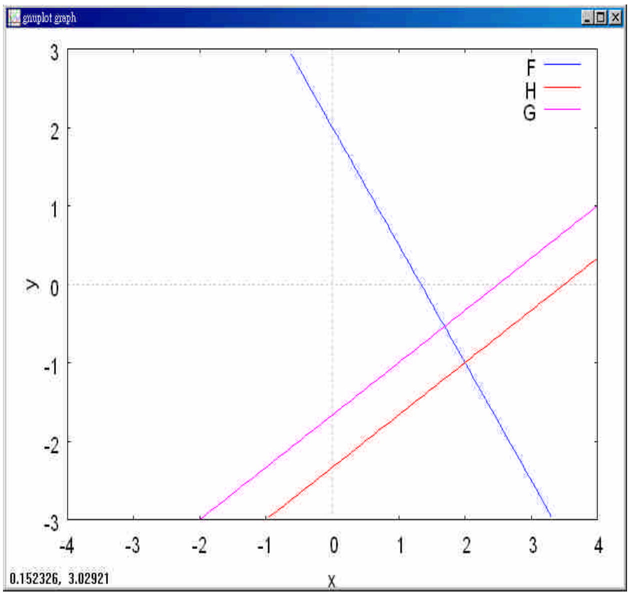
(%o57) $y1-\frac{3(x-x1)}{2}$

(%i60) f(x1,y1):=y1-(3/2)*(x-x1);

(%o60) $f(x1,y1):=y1-\frac{3}{2}(x-x1)$

(%i63) f(2,-1);

(%o63) $-\frac{3(x-2)}{2}-1$



(%i68) F(x):=(-3/2)*(x-2)-1;
H(x):=(2/3)*(x-2)-1;
G(x):=(2/3)*x-(5/3);

plot2d ([F,H,G],[x,-4,4],[y,-3,3]);

(%o68) $F(x):=\frac{-3}{2}(x-2)-1$

(%i69)

(%o69) $H(x):=\frac{2}{3}(x-2)-1$

(%o70) $G(x):=\frac{2}{3}x-\frac{5}{3}$

plot2d: some values were clipped.
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(%o71)