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Example 1

Sketch the graph of $y = x^2 - 2$

Sol:

First construct a table of values. Then plot the points shown in the table.

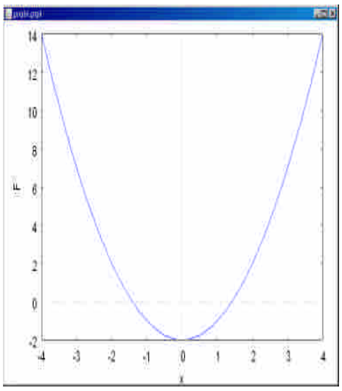
x	-2	-1	0	1	2	3
y	2	-1	-2	-1	2	7

Finally, connect the points with a smooth curve, as shown in Figure P.2. This graph is a **parabola**. It is one of the conics you will study in Chapter 10.

```
(%i1) F(x) := x^2-2;
      plot2d(F,[x,-4,4]);
```

(%o1) $F(x) := x^2 - 2$

(%o2)



Example 2

Find the x and y intercepts of the graph of $y = x^3 - 4x$

Sol:

To find the x intercepts, let y be zero and solve for x.

$x^3 - 4x = 0$ Let y be zero

$x(x-2)(x+2) = 0$ Factor

$x = 0,\ 2,\ or\ -2$ Solve for x

Because this equation has three solutions, you can conclude that the graph has three x intercepts:

(0,0), (2,0), and (-2,0). x intercepts

To find the y intercepts, let x be zero. Doing this produces y=0. So, the y intercept is (0,0) y intercept

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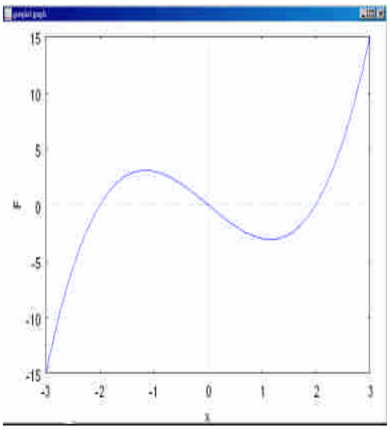
(%i3) solve([x^3-4*x=0],[x]);

(%o3) $[x=-2, x=2, x=0]$

(%i4) F(x) := x^3-4*x;
plot2d(F,[x,-3,3]);

(%o4) $F(x):=x^3-4\,x$

(%o5)



Example 3

Show that the graph of $y=2x^3-x$ is symmetric with respect to the origin

Sol:

$y=2x^3-x$	Write original equation
$-y=2(-x)^3-(-x)$	Replace x by –x and y by -y
$-y=-2x^3+x$	Simplify
$y=2x^3-x$	Equivalent equation

Because the replacements yield an equivalent equation, you can conclude that the graph of $y=2x^3-x$ is symmetric with respect to the origin, as shown in Figure P.8

(%i13) F(x) := 2*x^3-x;
plot2d(F,[x,-3,3]);

(%o13) $F(x):=2\,x^3-x$

(%o14)

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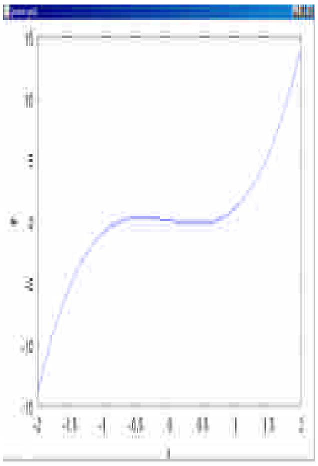


Example 4

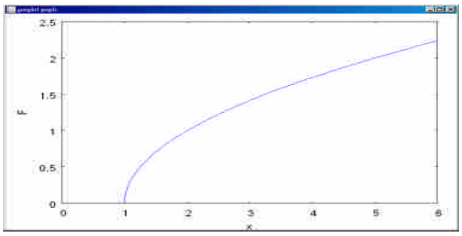
Sketch the graph of $x - y^2 = 1$

Sol:

The graph is symmetric with respect to the x axis because replacing y by –y yields an equivalent equation.

$x - y^2 = 1$	Write original equation
$x - (-y)^2 = 1$	Replace y by -y
$x - y^2 = 1$	Equivalent equation

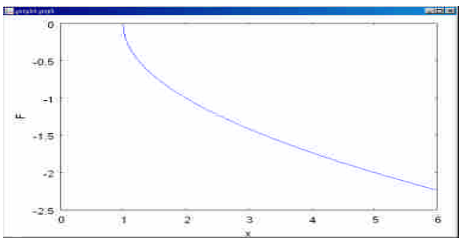
This means that the portion of the graph below the x axis is a mirror image of the portion above the x axis. Then reflect in the x axis to obtain the entire graph, as shown in Figure P.9.



(%i27) F(x) := sqrt(x-1)

plot2d(F,[x,0,6]);

(%o27) $F(x) := \sqrt{x-1}$



(%i29) F(x):=-sqrt(x-1);
plot2d(F,[x,0,6]);

(%o29) $F(x):=-\sqrt{x-1}$

plot2d: expression evaluates to non-numeric value somewhere in plotting range.

(%o30)

Example 5

Find all points of intersection of the graphs of $x^2 - y = 3$ and $x - y = 1$

Sol:

Begin by sketching the graphs of both equations on the same rectangular coordinate system, as shown in Figure P.10. Having done this, it appears that the graphs have two points of intersection. You can find these two points, as follows.

$y = x^2 - 3$	Solve first equation for y.
$y = x - 1$	Solve second equation for y.
$x^2 - 3 = x - 1$	Equate y values.
$x^2 - x - 2 = 0$	Write in general form.
$(x-2)(x+1) = 0$	Factor.
$x = 2 \ or \ -1$	Solve for x.

The corresponding values of y are obtained by substituting x=2 and x=-1 into either of the original equations. Doing this produces two points of intersection:

(2, 1) and (-1, -2) Points of intersection

(%i6) f(x):=x^2-3;

(%o6) $f(x):=x^2-3$

(%i7) f(-1);

(%o7) -2

(%i8) f(2);

(%o8) 1

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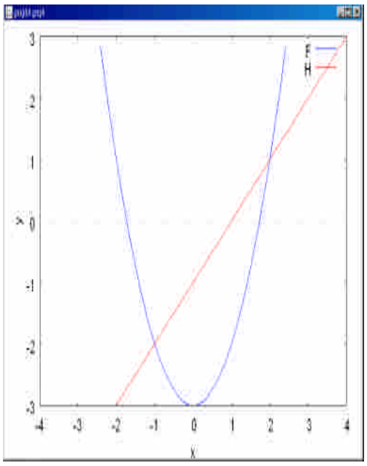


```
(%i62) F(x):=x^2-3;
       H(x):=x-1;
       plot2d ([F,H],[x,-4,4],[y,-3,3]);
```

(%o62) $F(x):=x^2-3$

(%o63) $H(x):=x-1$

Example 6

The Mauna Loa Observatory in Hawaii records the carbon dioxide concentration y(in parts per million) in Earth's atmosphere. The January readings for various years are shown in Figure P.11. In the July 1990 issue of Scientific American, these data were used to predict the carbon dioxide level in Earth's atmosphere in the year 2035, using the quadratic model

$$y = 316.2 + 0.70t + 0.018t^2$$

where t=0 represents 1960,as shown in Figure P.11(a).

The data shown in Figure P.11 (b) represent the years 1980 through 2002 and can be modeled by

$$y = 306.3 + 1.56t$$

where t=0 represents 1960. What was the prediction given in the Scientific American article in 1990? Given the new data for 1990 through 2002, does this prediction for the year 2035 seem accurate?

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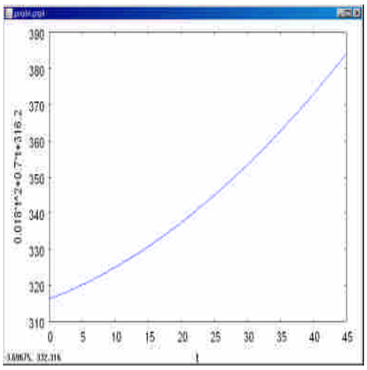

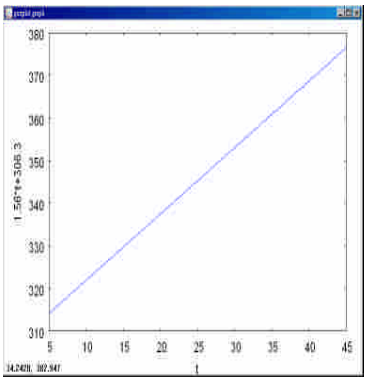

Figure P.11 (a) (b)

plot2d ([316.2+0.70*t+0.018*t^2],[t,0,45]); plot2d ([306.3+1.56*t],[t,5,45]);

Sol:

To answer the first question, substitute t=75 (for 2035) into the quadratic model.

$$y = 316.2 + 0.70(75) + 0.018(75)^2 = 469.95$$ Quadratic model

So, the prediction in the Scientific American article was that the carbon dioxide concentration in Earth's atmosphere would reach about 470 parts per million in the year 2035. Using the linear model for the 1980-2002 data, the prediction for the year 2035 is

$$y = 306.3 + 1.56(75) = 423.3$$ Linear model

So, based on the linear model for 1980-2002, it appears that the 1990 prediction was too high.

```
(%i1) f(t):=316.2+0.70*t+0.018*t^2;
(%o1) f(t):=316.2+0.7 t+0.018 t^2

(%i2) f(75);
(%o2) 469.95

(%i5) h(t):=306.3+1.56*t;
(%o5) h(t):=306.3+1.56 t

(%i6) h(75);
(%o6) 423.3
```